Background
Method
Solver
Elastic results
Plastic method
Plastic results

Element-free elastoplastic solid for nonsmooth multidomain dynamics

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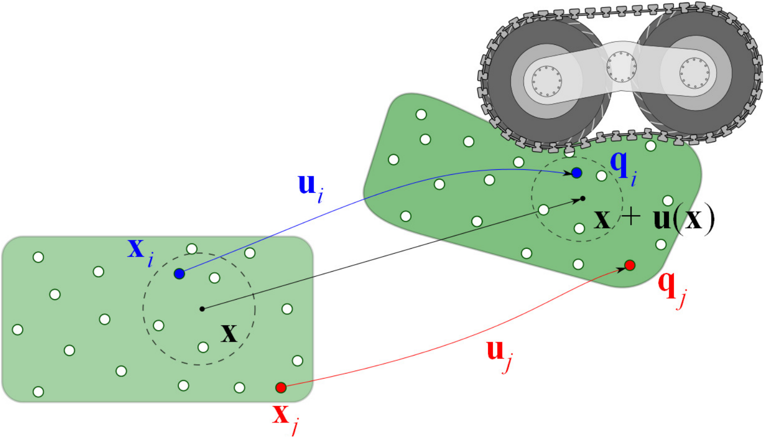
Approaches

Solid mechanics

$$\rho \ddot{u} - \nabla \cdot \sigma = \rho b$$
$$\sigma = CE$$

Multibody system dynamics

$$M \ddot{q} - G^T \lambda = f$$
$$\varepsilon \lambda + g(q) = 0$$

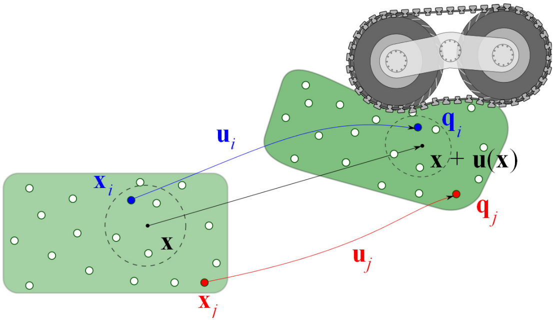

$\mathbf{u}_i$
$\mathbf{x}_i$
$\mathbf{q}_i$
$\mathbf{x} + \mathbf{u}(\mathbf{x})$
$\mathbf{x}$
$\mathbf{q}_j$
$\mathbf{u}_j$
$\mathbf{x}_j$

Displacement $u(x)$

$$q(x) = x + u(x)$$
$$J \equiv \nabla_x q(x) = I + \nabla_x u$$

Green-Lagrange strain tensor

$$E(x) = \frac{1}{2}\left(J^T J - I\right) = \frac{1}{2}\left(\nabla_x u + \nabla_x^T u + \nabla_x^T u \nabla_x u\right)$$



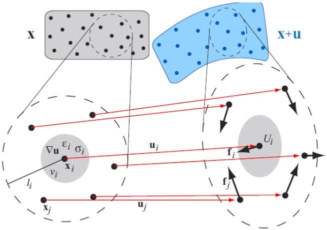
Moving least squares (MLS) approximation of the displacement field $u(\mathbf{x})$

$$u_\alpha(x) = \sum_{j}^{Np} \Psi_j(x) u_\alpha^j$$

$$\Psi_j(x) = p_\gamma(x) A_{\gamma\tau}^{-1}(x) p_\tau(x^j) W\left(\mathbf{x} - \mathbf{x}^j, h\right)$$

$$A_{\gamma\tau}(x) = \sum_{j}^{Np} W\left(x - x^j, h\right) p_\gamma(x^j) p_\tau(x^j)$$

$$p(x) = \left[1,\ x,\ y,\ z,\ yz,\ xz,\ xy,\ x^2,\ y^2,\ z^2\right]^T$$



Constraint energy

$$U = \frac{1}{2} g^T \varepsilon^{-1} g$$

Constitutive relation using Voigt notation

$$\sigma = CE$$

$$C = \begin{bmatrix} \lambda + 2\mu & \lambda & \lambda & 0 & 0 & 0 \\ \lambda & \lambda + 2\mu & \lambda & 0 & 0 & 0 \\ \lambda & \lambda & \lambda + 2\mu & 0 & 0 & 0 \\ 0 & 0 & 0 & \mu & 0 & 0 \\ 0 & 0 & 0 & 0 & \mu & 0 \\ 0 & 0 & 0 & 0 & 0 & \mu \end{bmatrix}$$

Strain energy

$$U = \frac{1}{2} E v_0 C E$$

Elasticity strain tensor constraint and regularisation - 6D

$$g^i(q) = E^i \qquad \varepsilon = (v_0 C)^{-1}$$

Jacobian where $K_{\tau\eta} \equiv \nabla_\tau u_\eta$

$$G^i_{\alpha\beta}(q) = \frac{\partial g^i_\alpha}{\partial q_\beta} = \frac{\partial g^i_\alpha}{\partial E_\gamma} \frac{\partial E_\gamma}{\partial K_{\tau\eta}} \frac{\partial K_{\tau\eta}}{\partial q_\beta}$$

Multibody dynamics - numerical solver

Linearized varational time stepper SPOOK (Lacoursière [2, 3])

$$q_{n+1} = q_n + h\dot{q}_{n+1}$$

$$\underbrace{\begin{bmatrix} M & -G^T & -\bar{G}^T \\ G & \Sigma & 0 \\ \bar{G} & 0 & \bar{\Sigma} \end{bmatrix}}_{H} \underbrace{\begin{pmatrix} \dot{q}_{n+1} \\ \lambda \\ \bar{\lambda} \end{pmatrix}}_{z} = \underbrace{\begin{pmatrix} M\dot{q}_n + hf_n \\ -\frac{4}{h}\Upsilon g + \Upsilon G\dot{q}_n \\ \omega_n \end{pmatrix}}_{-r}$$

regularization and stabilization matrices

$$\Sigma = \frac{4}{h^2}\,\mathrm{diag}\left(\frac{\varepsilon_1}{1+4\frac{\tau_1}{h}}, \frac{\varepsilon_2}{1+4\frac{\tau_2}{h}}, \ldots\right) \qquad \varepsilon = (\nu_0 C)^{-1}$$

$$\bar{\Sigma} = \frac{1}{h}\,\mathrm{diag}\left(\gamma_1, \gamma_2, \ldots\right)$$

$$\Upsilon = \mathrm{diag}\left(\frac{1}{1+4\frac{\tau_1}{h}}, \frac{1}{1+4\frac{\tau_2}{h}}, \ldots\right)$$

Nonsmooth MBD - numerical solver

Including frictional contacts, impacts, joint ant motor limits lead to limits and complementarity conditions on the solution variables

$$Hz + r = w_+ - w_-$$
$$0 \leqslant w_+ \perp z - l \geqslant 0$$
$$0 \leqslant w_- \perp u - z \geqslant 0$$

The problem transforms from linear system to a mixed linear complementarity problem (MLCP)

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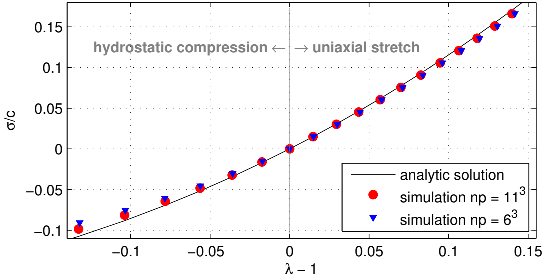
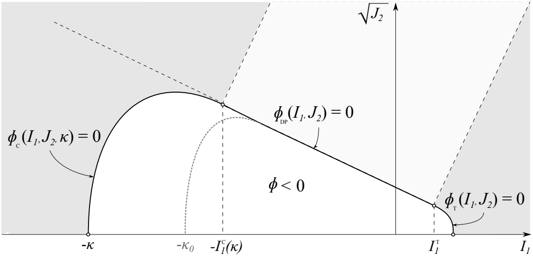


Elastoplastic terrain model

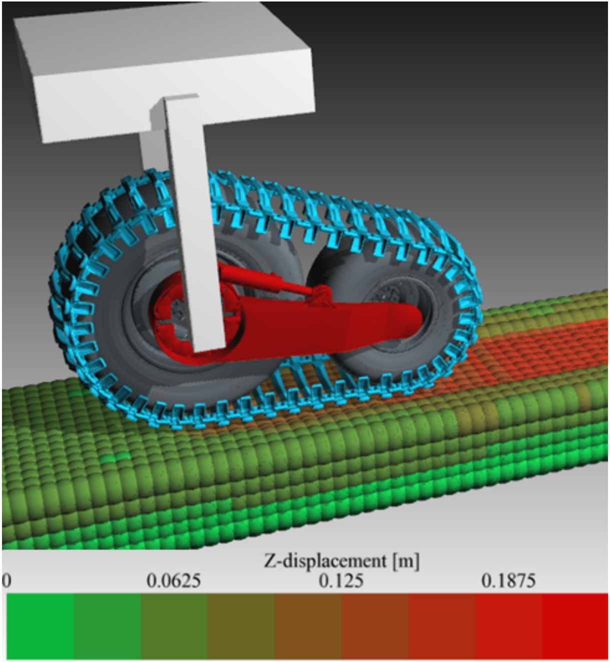
Elastic and plastic strain components $E = E^e + E^p$

Plastic flow rule $dE^p = d\lambda^p \frac{\partial \Phi}{\partial \sigma}$ when yield $\Phi(\sigma) > 0$



Capped Drucker-Prager plasticity model (Dolarevic [1])

$$\Phi\left(\sigma, \kappa\right) = \begin{cases} \Phi_t\left(I_1, J_2\right) & I_1 \geqslant I_1^t \\ \Phi_e\left(I_1, J_2\right) & I_1^t \leqslant I_1 \leqslant I_1^c\left(\kappa\right) \\ \Phi_c\left(I_1, J_2, \kappa\left(\mathrm{tr}\, E^p\right)\right) & I_1 \leqslant I_1^c\left(\kappa\right) \end{cases} \quad (1)$$


Z-displacement [m]
0
0.0625
0.125
0.1875

References

[1] S. Dolarevic et al. A modifed three-surface elasto-plastic cap model and its numerical implementation. Comput. Struct., 85(7-8):419-430, April 2007.

[2] C. Lacoursière, M. Linde, SPOOK: a variational time-stepping scheme for rigid multibody systems subject to dry frictional contacts, submitted (2013).

[3] C. Lacoursière, Ghosts and Machines: Regularized Variational Methods for Interactive Simulations of Multibodies with Dry Frictional Contacts, PhD thesis, UmeåUniversity, Sweden, (2007)

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Drucker-Prager for cohesive soil

$$\Phi_e(I_1, J_2) = \sqrt{J_2} + \eta(\phi)\frac{I_1}{3} - \xi(\phi)\,c \qquad (2)$$

Tension cap

$$\Phi_t(I_1, J_2) = (I_1 - T + R_t)^2 + J_2 - R_t^2 \qquad (3)$$

Compression cap

$$\Phi_c(I_1, J_2, \kappa(\mathrm{tr}\,\varepsilon^p)) = \frac{(I_1 - a(\kappa))^2}{R^2} + J_2 - b(\kappa)^2 \qquad (4)$$

Cap variables - for compressive hardening

$$\kappa = \kappa_0 + \frac{1}{D}\ln\left(1 + \frac{\mathrm{tr}(\epsilon^p)}{W}\right) \qquad (5)$$

where κ_0 is the initial position of the compression cap (Dolarevic [1]).