Adaptive Simulation of Coupled Flow-Transport Problems

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Abstract. We develop a computational framework for adaptive solution of coupled flow transport problems. The framework is based on a posteriori error estimation techniques which in particular capture the amplification of errors in the coupling of the flow and transport equations. The estimates are used to design efficient solution strategies involving automatic adaptive tuning of method parameters such as the mesh size and stopping criteria in the individual solvers.

Coupled flow-transport problems occur in many important applications including groundwater transport and heat transfer in fluids. In this work we introduce a systematic technique for solving such problems by coupling of a fluid solver and a transport solver. This is a standard way of constructing a multiphysics solver but the analysis of accuracy and stability of these methods remain largely open. There are also interesting issues concerning efficient solution methodology. We consider in particular applications to heat driven fluid flow in three dimensions.

The method is based on adaptive finite element solution of the Navier-Stokes equations followed by solution of the transport equation and builds on an a posteriori error estimate for the quantity of interest in the transport equation, for instance the integrated heat flux at a certain interface or a time integral of that quantity. The a posteriori error estimate is derived using duality techniques and the error in the goal quantity is estimated in terms of a residual and the L^2-norm of the error in the flux data coming from the Navier-Stokes equations weighted by certain dual weights. The L^2-error in the flux can then be estimated using an a posteriori error estimate for the Navier-Stokes equations. Thus the adaptivity is coupled and gives a systematic way of tuning discretization parameters, such as the mesh size and the stopping criteria for the iterative solvers, in the different solvers. The analysis is an extension of [1].

Finally, we describe the extension to simulation of combustion processes where a reaction diffusion equation modeling the chemistry of the combustion is added to the model. The combustion gives rise to significantly more complex phenomena involving moving thin layers.

References

1. M.G. Larson and A. Målqvist, Simulation of Oil Reservoirs Using Goal Oriented A Posteriori Error Estimates, preprint, www.math.umu.se/cm.