# Locating Cardiac Ischemia

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**Abstract.** The paper describes an optimization problem of identifying heart ischemia using ECG measurements on the torso surface. Heart ischemia is a condition of reduced blood supply to cardiac tissue and can lead to heart infarctions. In the simulations, we model the cardiac electrical activity by a stationary form of the bidomain equations. These are solved using an adaptive finite element method on a realistic geometrical model of the heart, lungs and torso, obtained from magnetic resonance imaging data. To model the ischemic region, we use a set of parameters in a level set framework. The ischemia is identified by minimizing a cost functional. Since this is a severly ill-posed problem, we investigate the effect of different regularization methods.

## 1 Introduction

Cardiovascular diseases are the leading cause of death for both men and women in the western world. In this paper, we study one kind of heart diseases, ischemic heart diseases. Ischemia is a condition of reduced blood supply to tissue, usually due to constriction or blocking of blood vessels. If such a condition persists, it will cause necrosis and cell death – an infarction. We present a method of locating ischemic regions in a realistic model of the heart by formulating an optimization problem.

During an ischemia, cells will have an altered electric behaviour. This effect can be seem on an electrocardiogram, ECG, which is the measurement of the electrical activity on the torso surface. The ECG is a fundamental tool for diagnosing cardiovascular diseases such as ischemia. Sometimes, however, only a crude picture of the position and size of an ischemia can be obtained, cf. Birnbaum and Drew [1]. In certain cases, the ECG may even fail to detect an ischemia, see for example Lau et. al. [2]. We investigate if simulating the cardiac electrical activity and solving the optimization problem may assist a physician in the task of locating an ischemia.

## 2 Mathematical Model

To formulate an optimization problem of finding the ischemic region given measurements on the torso surface, a model of the electrical activity in the heart and

the torso is needed. We use the most widely utilized model for the cardiac electrophysiology, the bidomain equations. The model was derived by Tung in 1978, see [4], and consists of a system of partial differential equations. A derivation is based on a representation of the tissue as an intracellular and extracellular media, both with aniostropic conductivity. Denoting the heart domain by $H$, the extracellular potential by $u_e$ and the transmembrane potential by $v$ the model reads

$$\nabla \cdot (M_i \nabla v) + \nabla \cdot (M_i \nabla u_e) = v_t + I_{ion}, \quad x \in H, \tag{1}$$
$$\nabla \cdot (M_i \nabla v) + \nabla \cdot ((M_i + M_e) \nabla u_e) = 0, \quad x \in H. \tag{2}$$

Here, $M_i$ and $M_e$ denotes the intracellular and extracellular conductivity, respectively.

A system of ordinary differential equations models the ionic transport $I_{ion}$ across the cell membranes, and these couple to the transmembrane potential in the bidomain model. Many different ion transport models are available, each taking different physiological phenomena into account. The torso, denoted by $T$, can be modeled as a passive conductor since it is assumed to consist of non-excitable cells without current sources. Hence, denoting the torso potential by $u_T$ and the conductivity by $M_T$ we have

$$\nabla M_T \cdot \nabla u_T = 0, \quad x \in T. \tag{3}$$

The principle of locating an ischemic region is based on the fact that such cells have reduced ability to excite. To introduce this effect to the model, the conductivities and the transmembrane potential in the bidomain model must be modified. This is done by introducing a set of parameters $p$ describing the ischemic region. Typically these represent the size and location of the ischemia. We use these parameters to form a level set representation of the infarction as presented by Lysaker and Nielsen in [3].

In this paper, the bidomain model is restricted to the ST-phase of the ECG, which is the segment of the ECG where the effects of an ischemia are most prominent. During this short period of the cardiac cycle, there is little effect of the ion transport and the full bidomain model can be reduced to a stationary model. Summarizing the mathematical model, we have

$$\nabla \cdot (M_i \nabla v) + \nabla \cdot (M_i \nabla u_e) = I_{ion}, \quad x \in H, \tag{4}$$
$$\nabla \cdot (M_i \nabla v) + \nabla \cdot ((M_i + M_e) \nabla u_e) = 0, \quad x \in H, \tag{5}$$
$$\nabla M_T \cdot \nabla u_T = 0, \quad x \in T. \tag{6}$$

For the boundary conditions, we require a continuous potential and a continuous current flow on the heart-torso boundary:

$$u_e = u_T, \quad x \in \partial H, \tag{7}$$
$$n_H \cdot M_e \nabla u_e + n_T \cdot M_T \nabla u_T = 0, \quad x \in \partial H. \tag{8}$$

We also require a zero normal component of the intracellular current, and a torso isolated from its surroundings:

$$n_H \cdot M_i \nabla(v + u_e) = 0, \quad x \in \partial H, \tag{9}$$

$$n_T \cdot M_T \nabla u_T = 0, \quad x \in \partial T. \tag{10}$$

The goal of the optimization procedure is to determine $p$ given ECG measurements $d$ on the surface of the torso. In order to do so, we introduce a cost functional

$$J(p) = \frac{1}{2} \int_{\partial T} (d(x) - u_T(x; p))^2 \ dx. \tag{11}$$

The problem is to fint $p$ such that $J = J(p)$ is minimized. Like many other problems of this kind, which are referred to as inverse problems, this can be severly ill-posed. Regularization methods must therefore be used to obtain stable solution approximations.

## 3 Implementation Details

To perform a realistic simulation, it is necessary to have a geometrical description of the heart and torso of high detail. Also, one must have access to features such as muscle fiber structures, since these are essential for modeling the strongly anisotropic conductivities. We have in our collaboration with Simula Research Laboratory in Norway obtained such data, as well as surface triangulations based on volumetric data from magnetic resonance images as depicted in figure 1. Using these tesselations of the torso, heart and lung surface, we form a volume triangulation using an octree based method. An adaptive finite element method
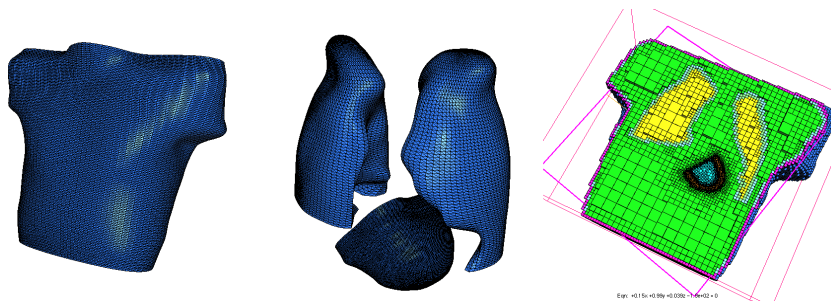


**Fig. 1.** Initial surface triangulations and a volume triangulation of hexahedra illustrating different regions

is used for solving the stationary bidomain equations. For finding the optimal set of paramaters describing the ischemia, these equations must be solved many times. Since the optimal set of meshes does not vary much for each time $p$

is updated, the computation of error indicators are not needed in every step. Hence, the overhead of using an adaptive setup is small. Also, with an octree based geometrical setup used, seperate meshes for the dual and primal problem can be handled.

Different techniques are available for solving the minimization problem, but we follow Lysaker and Nielsen [3] and use a standard gradient-based method. Particularly, they show how to compute all the gradients by solving a single adjoint problem. We extend the work by considering adaptivity for this problem.

## References

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