Numerical Simulation of 2D Radiation Heat Transfer Problem

S.V. Polyakov, T.A. Kudryashova, A. Kononov, A. Sverdlin

Institute for Mathematical Modeling RAS, 125047, Miusskaya sq. 4A, Moscow, Russia
kudryashova@imamod.ru

Abstract. This is a highly relevant subject because planetary missions have been launched and indeed some of them are currently in progress such as Spirit or Dnepr and others. The design of a thermal protection system for the space vehicle is an important problem. It requires accurate prediction of the heating environments. This study uses diffusion approximation to investigate radiation heat transfer for reentry vehicles. The goals of the paper to present a parallel algorithm: to calculate a strongly radiating axisymmetric flow field over a blunt body; to introduce a database of absorption coefficients and use it in parallel computing; to compare results of flow with and without radiation influence.

1 Introduction

The purpose of this paper is the development of parallel numerical algorithms for radiation gas dynamics problems adapted to architecture of multiprocessor systems with distributed memory. Investigations of flows over solid body coming into atmosphere is traditionally important direction of high temperature gas dynamics. The work is devoted the simulation of radiation heat transfer for reentry vehicles. Radiation heat flux become significant during the reentry of vehicles into Earth's atmosphere. When the space vehicle flies at Mach number a shock wave is formed ahead of flying body. High temperature appears behind the shock wave and creates strong radiation. In such process radiation affects flow field and flow field affects radiation properties. Mutual influence means the join computing of the equation of energy for substance and the equation of radiation transport. For the investigations the model of diffusion approximation [1] has been used. The model was coupled with the gas dynamical part by the including the radiation flux in the energy equation. The quasigasdynamics equations (QGD) [2] were performed to compute gas dynamics flows. A radiation database has been developed in order to use the values of the absorption coefficients for problems like that. The experimental data of the absorption coefficients for database have been taken from [4]. The measurements of radiation are presented for full spectrum. The radiation heat transfer problem is computed for two dimensional axisymmetric flow around of the blunt body [5]. The calculations were carried out on rectangular grids. The mesh construction methods were developed in [6].

2 Numerical Procedure

In the model of diffusion approximation, the radiation transfer equation is replaced by two equations: the exact continuity equation for the radiation and the approximate equation connecting the flux and the density of the radiation. The second equation is obtained with assumption of the angular isotropy of the radiations field. The system of "diffusion equations" must be evaluated many times for the different wavelength. The number of the wavelength points needed varies with the radiating species in the flowfield. The "diffusion approximation" model was coupled with the gas dynamical part by the including the radiation flux in the energy equation. The objective of the present work is the developing of the stable and converging method for calculating a strongly radiating axisymmetric flowfield over a blunt body. The system of the quasi gas dynamical equations (QGD) is used for the flowfield calculations [3]. With the usual notations, the general form of QGD system is written as:

$$\frac{\partial}{\partial t}\rho + \nabla_i \rho J^i = 0 \tag{1}$$

$$\frac{\partial}{\partial t}\rho u^k + \nabla_i J^i u^k + \nabla^k p = \nabla_i \Pi^{ik} \tag{2}$$

$$\frac{\partial}{\partial t}E + \nabla_i \frac{J^i}{\rho}(E+p) + \nabla_i q^i = \nabla_i(\Pi^{ik}u_k) \tag{3}$$

with $E = 0.5\rho u^i u_j + \varepsilon$. Here ρ is the density, $\overrightarrow{u}$ the velocity vector, $p = \rho RT$ the pressure, R is the perfect gas constant, T is the temperature, $\varepsilon = p/(\rho(\gamma-1))$ the internal energy, γ is the specific heat ratio. The system (1) - (3) can be closed by the following relation for the mass flux vector J^i , the shear-stress tensor Π^{ik}, and the heat flux vector q^i:

$$J^i = J^i_{NS} - \tau(\nabla_j(\rho u^i u^j) + \nabla^i p), J^i_{NS} = \rho u^i \tag{4}$$

$$\begin{aligned} \Pi^{ik} &= \Pi^{ik}_{NS} + \tau(u^i(\rho u^j \nabla_j u^k + \nabla^k p) + g^{ik}(u_j \nabla^j p + \gamma p \nabla_j u^j), \\ \Pi^{ik}_{NS} &= \mu(\nabla^k u^k + \nabla^i u^k - (2/3) g^{ik} \nabla_j u^j) + \eta g^{ik} \nabla_j u^j \end{aligned} \tag{5}$$

$$q^i = q^{ik}_{NS} - \tau\rho u^i(u^j \nabla_j \varepsilon + p u_j \nabla^j (1/\rho)), q^{ik}_{NS} = -k\nabla^i T \tag{6}$$

here the coefficients μ and κ are the viscosity and heat conductivity coefficient respectively, g^{ik} is the metric tensor, τ is the Maxwell's relaxation time, equal to $\tau = \mu/p$.

QGD equations are obtained by a kinetical approach. The Boltzman equation is integrated with collisional invariants, using a special variant of serial expansion for the distribution function [2]. For the stationary flow, the dissipative terms (terms in τ) in the QGD equations have the asymptotic order of $O(Kn^2)$ for $Kn \longrightarrow 0$.

$$div\overrightarrow{W_\nu} + \chi_\nu c U_\nu = \chi_\nu c U_{\nu p} \tag{7}$$

$$\frac{c}{3} gradU\nu + \chi_\nu \overrightarrow{W_\nu} = 0 \tag{8}$$

Here $\overrightarrow{W_\nu}$ is the radiation flux for the frequency ν or for the wavelength $\lambda = c/\nu$, c is the light velocity, χ_ν is the absorption coefficient for the frequency ν, U_ν is the spectral density of radiation and $U_{\nu p}$ is the spectral density of the equilibrium emission. We have considered the hypotheses of the local thermodynamical equilibrium. Under this assumption the absorption coefficient at any given frequency is related to the emission coefficient via the black-body function evaluated at the local temperature. The diffusion equations (7) - (8) are evaluated for some number of the frequency groups. The total radiation flux is obtained as the sum of the radiation fluxes for each frequency in the form: $\sum_\nu \overrightarrow{W_\nu}$. As the last step the energy equation (3) is recalculated in the form:

$$\frac{\partial E}{\partial t} = div\overrightarrow{W} \tag{9}$$

Using this procedure we take into account the influence of the radiation upon the gas dynamic flowfield.

The problem of the gas dynamic flowfield calculation affected by the radiation may be divided into three subproblems.

The solution of the gas dynamic block (equations (1) - (6)) presents the first problem. The QGD equations are solved by means of an explicit algorithm. The geometrical parallelism principle has been implemented for the parallel realization. The computational domain is divided in the subdomains in z-direction and each processor provides calculations in its own subdomain. Note that in this stage of calculations we obtain the value of the total energy for the intermediate step in the time and we will evaluate this value in the third stage taking into account the influence of the radiation.

As the second step, the system of the "diffusion equations" (equations (7)-(8)) are solved for ten frequency groups. In this part we can use parallelization in accordance with radiation frequencies groups and a parallel conjugate gradient method is implemented to obtain the radiaton flux. The values of the absorption coefficients have been taken from [4].

In the third stage, the equation (9) is solved in order to taken into account the influence of the radiation to the gas dynamical flowfield. The domain decomposition technique is used in this part of the calculations.

The calculations were carried out with flow conditions. The parameters of flow are reported in the Table 1.

3 Database

For calculations the database has been used. The radiation database has been developed in order to use the values of the absorption coefficients for problems like that.

The diffusion equations were evaluated for 600 frequency groups. Spectral interval is $250cm^{-1} - 150000cm^{-1}$. In the database the spectral step is $250cm^{-1}$.

Table 1. The initial conditions of the free stream

Background gas temperature $T_0(K)$	266
Temperature of head shield $T_1(K)$	$2000 \div 3300$
Pressure $p_0(Pa)$	4.3*10
Density $\rho_0(kg/m^3)$	$5.63 * 10^{-4}$
Characteristic size of problem Ł(m)	$5.035 * 10^{-1}$
Mach number M_∞	12,24

Temperature step is 500K. Database supports approximately 10^5 meanings of the absorption coefficients. We can call data from database and use them in our parallel code. Well known limitations for values of the absorption coefficients for diffusion approach are checked

$$\frac{l_\nu}{L} \ll 1, \qquad l_\nu = \frac{1}{\chi_\nu}$$

here L is characteristic size of the problem, χ_ν - absorption coefficient.

Notes and Comments. Numerical method has been realized as a FORTRAN parallel code. The homogeneous parallel computer system MVS-5000 equipped with 336 - PowerPC970 (1.6GHz) microprocessors was employed. The total performance is over 2.1 Tflops and the fast communication links give up to 340 MB/sec (peak speed) data transmission rate.

This work was supported by Russian Foundation for Basic Research (grants No. 06-01-00233, 05-07-90230)

References

1. B.N. Chetverushkin: Dynamics of radiating gas. Nauka, Moscow (1980) (in Russian).
2. T.G.Elizarova, Yu.V. Sheretov: (2001) Theoretical and numerical investigation of quasigasdynamics and quasihydrodynamics equations, Comput. Mathem. And Mathem. Phys., **41**, (2001) 219–234.
3. T.G.Elizarova, M. Sokolova Numerical algorithm for computing of supersonic flows is based on quasi-hydrodynamics equations. Vestnik of Moscow State University, Phisics & Astronomy, (2003) (in Russian).
4. Avilova I.V., Biberman L.M., Vorob'ev V.S., Zamalin V.M., Kobzev G.A.: The optical characteristics of hot air. Moscow, Nauka, 320 p. (in Russian), (1970).
5. Sakai T, Tsuru T., Sawada K.: Computation of Hypersonic Radiating Flowfield over a Blunt Body. In Journal of Thermophysics and Heat Transfer. **15**(1) 91–98, (2001).
6. I.V. Popov,S.V. Polyakov: Construction of adaptive irregular triangular grids for 2D multiconnected non-convex domains. Mathematical modeling,**14**(6) 25–35, (2002) (in Russian).