StratiGraph and the Matrix Canonical Structure Toolbox

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TOOLS FOR COMPUTING STRUCTURAL INFORMATION

Study of canonical forms for

- Matrices, matrices of bilinear forms
- Matrix pencils $\boldsymbol{G} - \lambda \boldsymbol{H}$ (singular, regular, (skew-)symmetric)
- System pencils associated with generalized state-space systems

$$\boldsymbol{E\dot{x} = Ax + Bu}, \quad \boldsymbol{y = Cx + Du}$$

- Linearizations of polynomial matrices $\boldsymbol{P(s) = P_d s^d + \cdots + P_1 s + P_0}$

Motivation

- Determining the dynamics and system characteristics (like poles, zeros, controllability)
  - ▷ Understand how small changes in the system matrices, e.g., due to round-off errors or external disturbances, may affect the system characteristics

Ill-posed problems!

- Small perturbations of data matrices can drastically change the computed structural information

Qualitative information from orbit and bundle Stratifications

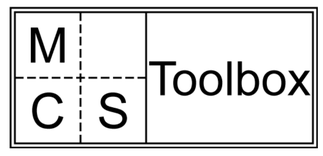
- Provides information for a deeper understanding how system transitions can take place under small perturbations
- Reveals the closure hierarchy of orbits and bundles

⇒ *StratiGraph*

Quantitative information

- Staircase forms: reveals the canonical structure information
- Upper and lower bounds to nearby canonical structures
- Distance to uncontrollability

⇒ *Matrix Canonical Structure Toolbox* for Matlab



STRATIGRAPH

StratiGraph* is a Java-based tool to compute, view, and investigate qualitative information on the relation between different canonical structures of an input setup.

The stratification reveals the closure hierarchy of obits (or bundles) of the setup, where all, e.g., matrices, in an orbit have the same canonical form.

Supported problem setups (new in version 3.1)

- Matrices under similarity
- Matrix pencils $\boldsymbol{G} - \lambda \boldsymbol{H}$ under strict equivalence
- System pencils

$$\begin{bmatrix} \boldsymbol{A} & \boldsymbol{B} \\ \boldsymbol{C} & \boldsymbol{D} \end{bmatrix} - s \begin{bmatrix} \boldsymbol{E} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} \end{bmatrix}, \quad \boldsymbol{\det(E) \neq 0}$$

  - ▷ Controllability pairs (A,B) under feedback equivalence
  - ▷ Observability pairs (A,C) under injection equivalence
- Polynomial matrices of full normal rank

Other features

- Export/import setups to/from Matlab
- The canonical structure information can be shown in different notations
- Expand whole or part of the graph (e.g., around a canonical structure of interest)
- Nodes can interactively be moved horizontally and deleted
- Expandable with new setups and extensions using plug-ins
- OS independent

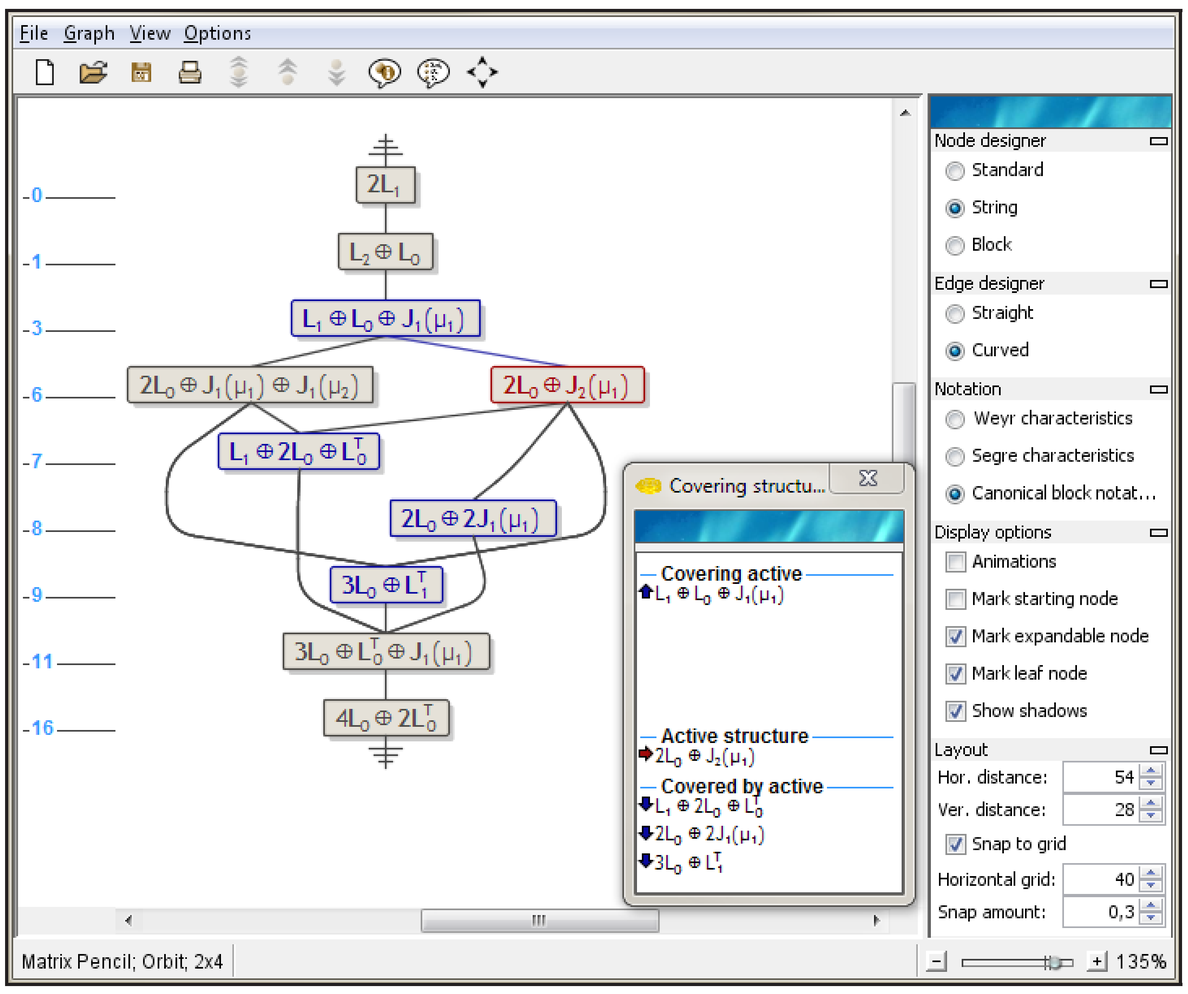


Figure: StratiGraph: Closure hierarchy graph—nodes are orbits, edges show covering relations (nearest neighbors) between orbits

* StratiGraph is an acronym for *Strati*fication *Graph*s.

MATRIX CANONICAL STRUCTURE TOOLBOX

The Matrix Canonical Structure (MCS) Toolbox is a Matlab toolbox including both computational routines and a framework for representing canonical structure objects.

The canonical structure is defined by the regular structures (finite and infinite eigenvalues together with their geometric/algebraic multiplicities) and the singular structures (right/left minimal indices), depending on which are applicable.

Data type objects exist for representing canonical structures of

- matrices (under similarity, congruence, and *congruence)
- matrix pencils (under strict equivalence, and symmetric and skew-symmetric pencils under congruence)
- system pencils (under feedback-injection equivalence)

Numerical routines exist for computing

- the tangent space
- the codimension, either from matrices or from the canonical structure information
- the staircase form (the Guptri form) of matrices and matrix pencils (*under development*)

POSSIBLE FUTURE EXTENSIONS

- General and structured polynomial matrices
- Generalized state-space systems (descriptor systems)
- Upper and lower bounds of distances between canonical structures
- Staircase forms for more supported canonical structure objects
- and more...

FURTHER INFORMATION

For references, contact information, and download see

http://www.cs.umu.se/english/research/groups/matrix-computations/stratigraph/

EXTERNAL FUNDING

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